Simply connected surfaces of general type with $p_g = 0$ and $K^2 = 2$

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Abstract. One of the fundamental problems in the classification of complex surfaces is to find a new family of simply connected surfaces with $p_g = 0$ and $K^2 > 0$. It has been studied intensively by algebraic surface theorists for more than 100 years since Enrique constructed a new surface with $p_g = q = 0$ and $\pi_1 = \mathbf{Z}_2$ in 1894. Although a large number of non-simply connected complex surfaces of general type with $p_g = 0$ and $K^2 > 0$ have been known ([BHPV], Chapter VII), until now the only previously known simply connected, minimal, complex surface of general type with $p_g = 0$ and $K^2 > 0$ was Barlow surface. Barlow surface has $K^2 = 1$. The natural question arises if there is a simply connected surface of general type with $p_g = 0$ and $K^2 \geq 2$.

Recently, I constructed a new simply connected symplectic 4-manifold with $p_g = 0$ (equivalently $b_2^+ = 1$) and $K^2 = 2$ by using a rational blow-down surgery [P]. After this construction, it has been a very intriguing question whether such a symplectic 4-manifold admit a complex structure.

The aim of this talk is to confirm an affirmative answer for the question above. Precisely, we construct a new family of simply connected, minimal, complex surfaces of general type with $p_g = 0$ and $K^2 = 2$ by modifying Park's symplectic 4-manifold in [P]. Our main techniques are a rational blow-down surgery and a $\mathbf{Q}$-Gorenstein smoothing theory. We also construct a family of simply connected, minimal, complex surfaces of general type with $p_g = 0$ and $K^2 = 1$ using the same technique.

In this talk, I'll sketch how to construct such 4-manifolds using a rational blow-down surgery and how to show that such 4-manifolds admit a complex structure using a $\mathbf{Q}$-Gorenstein smoothing theory.

References

[BHPV] W. Barth, K. Hulek, C. Peters, A. Van de Ven, *Compact complex surfaces*, 2nd ed. Springer-Verlag, Berlin, 2004.

[P] J. Park, *Simply connected symplectic 4-manifolds with $b_2^+ = 1$ and $c_1^2 = 2$*, Invent. Math. **1**59 (2005), 657–667.