

All tunnels of all tunnel number 1 knots

Darryl McCullough
University of Oklahoma, USA

Abstract. This is joint work with Sangbum Cho. We present a new theory which describes the collection of all tunnels of all tunnel number 1 knots in the 3-sphere (up to orientation-preserving equivalence in the sense of Heegaard splittings) using the disk complex of the genus-2 handlebody and associated structures. It shows that each knot tunnel is obtained from the tunnel of the trivial knot by a uniquely determined sequence of simple cabling constructions. A cabling construction is determined by a single rational parameter, so there is a corresponding numerical parameterization of all tunnels by sequences of such parameters and some additional data. Up to superficial differences in definition, the final parameter of this sequence is the Scharlemann-Thompson invariant of the tunnel, and the other parameters are the Scharlemann-Thompson invariants of the intermediate tunnels produced by the constructions. The theory extends easily to links, and adapts with little change to allow equivalence of tunnels by homeomorphisms that may be orientation-reversing. We discuss various applications and calculations.