Distance of knots and Morimoto's Conjecture on the super additive phenomena of tunnel numbers of knots

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Abstract. This is a joint work with Yo'av Rieck. Let K_i $(i = 1, 2)$ be knots in the 3-sphere S^3, and $K_1 \# K_2$ their connected sum. We use the notation $t(\cdot)$ to denote tunnel number of a knot. It is well known that the following inequality holds in general.

$$t(K_1 \# K_2) \leq t(K_1) + t(K_2) + 1.$$

We say that a knot K in a closed orientable manifold M admits a (g, n) position if there exists a genus g Heegaard surface which separates M into handlebodies H_1 and H_2, so that $H_i \cap K$ $(i = 1, 2)$ consists of n arcs that are simultaneously parallel into ∂H_i. It is known that if K_i $(i = 1$ or $2)$ admits a $(t(K_i), 1)$ position then the equality does not hold in the above. Morimoto proved that if K_1 and K_2 are m-small knots then the converse holds, and conjectured that this is true in general (K.Morimoto, Math. Ann., 317(3):489–508, 2000). Morimoto's Conjecture: Given knots $K_1, K_2 \subset S^3$, $t(E(K_1 \# K_2)) < t(E(K_1)) + t(E(K_2)) + 1$ if and only if for $i = 1$ or $i = 2$, K_i admits a $(t(K_i), 1)$ position.

In this talk, we describe how to show the existence of conterexamples to this conjecture by making use of 'distance' of knots.