Inverse limits of tent maps

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Abstract. Inverse limits are important in the study of dynamical systems.

Robert Williams showed that attractors for dynamical systems can be represented as inverse limits of branched manifolds with a single bonding map. It is natural to study and attempt to classify these inverse limits.

This has led to the problem of classifying the inverse limits of the form (I, f_s) where f_s is a member of the tent family, $f_s(x) = \min\{s \cdot x, s \cdot (1-x)\}$ $1 \leq s \leq 2$, and $I = [0, 1]$. It is conjectured that the inverse limits are homeomorphic if and only if $s = t$. This is known as Ingram's Conjecture. Special cases are known, but the general problem remains elusive. One special case stands out. Suppose that f_s and f_t both have periodic turning point. In this case Lois Kailhofer has shown that (I, f_s) is homeomorphic to (I, f_t) if and only if $s = t$.

The proof of Kailhofer's result is complicated, but a shorter and more transparent proof is given by Louis Block, Slagjana Jakimovic, Lois Kailhofer, and James Keesling. The new proof also shows that some power of any homeomorphism is isotopic to a power of the shift map on the inverse limit space.

The techniques developed in the new proof of Kailhofer's theorem suggest an approach to a general solution to Ingram's Conjecture. We will discuss the progress being made using this general approach.

Recent work by Brian Raines and Sonja Štimac has shown that for the class of tent maps with non-recurring turning point Ingram's Conjecture holds. It has been recently shown by Block, Keesling, and Misiurewicz that there are values of s for which the closure of the orbit of the turning point of f_s is an adding machine. This is the simplest case for which the turning point is recurrent, but not periodic. Progress in proving Ingram's Conjecture for this case will be discussed.

Related results by Christoph Bandt, Marcy Barge, Beverly Diamond, Karen Brucks, Henk Bruin, Brian Raines, Chris Good, Sonja Štimac, and others will also be reviewed.