Finiteness conditions and PD_r-group covers of PD_n-complexes

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Abstract. Gottlieb and Quinn showed that the homotopy fibre of a fibration $p : M \to B$ of a PD_n-complex M over a PD_r-complex B is a PD_{n-r}-complex if and only if it is finitely dominated. When B is aspherical and $p_* = \pi_1(p)$ is an epimorphism the homotopy fibre is the covering space corresponding to $\mathrm{Ker}(p_*)$. We shall show that in this case we may use duality to relax the hypothesis that the fibre be finitely dominated, to requiring merely that it be homotopy equivalent to a complex with finite $[n/2]$-skeleton. In the simplest nontrivial case, when the base is S^1, we can improve this slightly, and our result is then best possible. In particular, we obtain the following 4-dimensional homotopy analogue of the Stallings fibration theorem for 3-manifolds: *if M' is an infinite cyclic covering space of a closed 4-manifold M then M' satisfies Poincaré duality with local coefficients if and only if $\chi(M) = 0$ and $\pi_1(M')$ is finitely generated.* This is joint work with D.H.Kochloukova. Our argument is entirely homological, rather than homotopy-theoretic, and an essential element is her result that certain Novikov extensions of group rings are weakly finite.