

Handbook of

Conference on Explicit Birational Geometry

May 28th to June 1st, 2025

Peking University



Conference on Explicit Birational Geometry

The motivation for organizing this conference is the rapid development of explicit birational geometry in recent years—not only in terms of results but also through new methodologies and open problems. The primary goal of the conference is to bring together researchers working on explicit birational geometry from various perspectives.

Time and Venue:

Time: Wednesday, May 28th to Sunday, June 1st, 2025 Venue: Dingshisun Lecture Hall(丁石孙教室), Zhihua Building(智华楼), School of Mathematical Science, Peking University, No. 5 Yiheyuan Road, Haidian District, Beijing, China

Invited speakers:

Florin Ambro (IMAR) Fabrizio Catanese (Universität Bayreuth) Jungkai Chen (Taiwan University) Meng Chen (Fudan University) Jingjun Han (Fudan University) Chen Jiang (Fudan University) Zhi Jiang Fudan University) Masayuki Kawakita (RIMS, Kyoto University) ***=online talk** JongHae Keum (KIAS) Justin Lacini (Princeton University) Haidong Liu (Sun Yat-Sen University) Wenfei Liu (Xiamen University) Xin Lü (East China Normal University) Aleksandr Pukhlikov* (University of Liverpool) Miles Reid FRS (University of Warwick) Tong Zhang (East China Normal University)

Sponsers:

School of Mathematical Sciences, Peking University Beijing International Center for Mathematical Research, Peking University National Key Research and Development Program of China

Contact:

Jihao Liu (liujihao@math.pku.edu.cn)

Registration

For more details, please refer to the official conference website: <u>https://www.math.pku.edu.cn/kxyj/xzky/164429.htm</u>

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Time Schedule

	5/28 (Wed)	5/29 (Thu)	5/30 (Fri)	5/31 (Sat)	6/1 (Sun)
9:00-9:50	Registration				
9:50-10:00	Opening				
10:00-11:00	Fabrizio Catanese	Miles Reid FRS	Meng Chen	Jungkai Chen	JongHae Keum
	Group photo				
11:00-11:30	Tea Break				
11:30-12:30	Tong Zhang	Florin Ambro	Jingjun Han	Zhi Jiang	Wenfei Liu
12:30-14:00	Lunch Break ¹		Problem	Lunch Break ¹	Lunch ¹
14:00-15:00	Xin Lü	Chen Jiang	Session or free Afternoon	Masayuki Kawakita	Conference Concludes
15:00-15:30	Tea Break			Tea Break	
15:30-16:30	Justin Lacini	Aleksandr Pukhlikov ²		Haidong Liu	
18:00-20:00		Banquet ³			
	Day 1 Concludes	Day 2 Concludes	Day 3 Concludes	Day 4 Concludes	

1= Western Restaurant at Zhongguanxinyuan, Floor 2 of Zhongguanxinyuan building 1.

2= Online talk

3= Guanhu Room at Zhongguanxinyuan, Floor 2 of Zhongguanxinyuan building 1.

All talks in Dingshisun Lecture Hall (丁石孙教室), Floor 1 of Zhihua Building (智华楼), Peking University (北京大学).



Title and abstracts

Florin Ambro (IMAR)

Title: Classification of toric surface singularities

Abstract: In the search of minimal models, singularities appear inevitably. Explicit classification of these singularities is possible in small dimension, with important global applications. In higher dimension, explicit classification is considered hopeless, expecting instead qualitative results on invariants of singularities, such as the minimal log discrepancy, or the index of complements. I will survey the above in the first half of the talk. In the latter half, I will discuss a new qualitative classification of toric surface singularities.

Fabrizio Catanese (Universität Bayreuth)

Title: Fake homology quadrics, Surfaces isogenous to a Product, and numerically and cohomologically trivial automorphisms of Product quotient surfaces.

Abstract: Hirzebruch asked about the existence of even fake quadrics, that is, surfaces homeomorphic to $F_0 = P^{1} x P^{1}$ which are not rational (hence of general type). Similarly, odd fake quadrics would be minimal surfaces homeomorphic to the blow up F_1 of the plane P^{2} in one point.

Fake homology quadrics are the minimal surfaces of general type with the same Q-homology as F_0, F_1, namely the minimal surfaces with $K^2=8$, $p_g=q=0$.

It is conjectured that these are exactly the surfaces whose universal covering is the bidisk. There are 4 types: the surfaces isogenous to a product (these are a quotient of a product of curves) and the irreducible quotients, and, in each class, those with even, respectively odd intersection form.

I will show that the class of fake odd Q-homology quadrics is non empty, using surfaces isogenous to a product.

Fake quadrics isogenous to a product were classified in my joint work with Bauer and Grunewald (with a missing case added by Frapporti) and I will then speak of joint work in progress with Frapporti, showing that for the unmixed type the order $|Aut_Q|$ of the group of automorphisms acting trivially on Q-cohomology is at most 192, and equality is attained exactly for $G = (Z/2)^3$. The group Aut_Z of the automorphisms acting trivially on Z-cohomology is trivial, except possibly in one case where it could be of order 2.

I will then describe joint results with Wenfei Liu and Matthias Schuett, showing that for isotrivial properly elliptic surfaces one has the upper bound $|Aut_Z| \leq 3$, and there are infinitely many examples with such groups of order 2,3.

While there is no absolute bound for $|Aut_Q|$ if $p_g=q=0$, and we have examples attaining the maximum in the inequality $|Aut_Q| | |q | 4s | |q | (P_2 + 1) | (s is the number of singular fibres).$

Jungkai Chen (Taiwan University)

Title: Minimal models of threefolds of general type with small invariants

Abstract: In this talk, we are going to explicitly describe the minimal models of threefolds of general type on the Noether line or near the Noether line when the genus is small. We will demonstrate the framework of explicit resolution. Its application to the extremal cases leads to an explicit descripton of minimal models. This is a joint work in progress with Hsin-Ku Chen.



Meng Chen (Fudan University)

Title: The Noether inequality for algebraic 3-folds

Abstract: In this talk, I will present a complete proof for the following theorem: the inequality $K^3 \ge 4/3p_g-10/3$ holds for all 3-folds of general type.

Jingjun Han (Fudan University)

Title: Boundedness in general type minimal model programs

Abstract: One of the main open problems in the minimal model program is termination: any sequence of steps of a minimal model program terminates. It is hence interesting to further ask whether the number of steps in a minimal model program is bounded and whether an explicit bound can be computed. Motivated by local volumes introduced by Chi Li, we introduce a new local invariant "log canonical volume", which is non-decreasing in any sequence of MMP for general type varieties. As a result, in such kind of MMP, we show that (1) the Cartier index of any Weil Q-Cartier is uniformly bounded; (2) every fiber of the extremal contractions or the flips is bounded (3) the set of minimal log discrepancies belongs to a finite set. For general type varieties, we also show the termination of MMPs for general type varieties in dimension 5 and explicit termination in dimension at most 4. Joint work with Jihao Liu, Lu Qi, and Ziquan Zhuang.

Chen Jiang (Fudan University)

Title: Characterization of canonical threefolds with small genera and minimal volumes **Abstract:** For a smooth projective threefold of general type with geometric genus 2, it is known that its canonical volume is at lease 1/3. We will give a characterization of the equality case. It turns out that the canonical model of such a 3-fold must be a hypersurface of degree 16 in the weighted projective space P(1,1,2,3,8), which gives an explicit description of its canonical ring. This implies that the coarse moduli space parametrizing all canonical \$3\$-folds with canonical volume 1/3 and geometric genus 2, is an irreducible variety of dimension 189. This is a joint work with Meng Chen and Yong Hu.

Zhi Jiang (Fudan University)

Title: The canonical volumes of irregular threefolds of general type

Abstract: The canonical volume is probably one of the most important birational invariants of a smooth/canonical projective variety of general type. However, starting from dimension n>2, it is very difficult to determine the minimal canonical value v(n) of smooth projective varieties of general type of dimension n. In this talk, we will discuss the possible minimal canonical volume of irregular threefold of general type. This is a joint work in progress with Jungkai Chen, Yong Hu and Tong Zhang.

Masayuki Kawakita (RIMS, Kyoto University)

Title: Minimal log discrepancies of a fixed threefold

Abstract: The minimal log discrepancy is an invariant of singularities related to termination of flips. The ACC for minimal log discrepancies is still unknown in dimension three, and it is one of the most important remaining problems in the minimal model theory of threefolds. I will explain a proof of the ACC for minimal log discrepancies on an arbitrary fixed threefold. It uses a classification of threefold divisorial contractions.



JongHae Keum (KIAS)

Title: Combinatorially minimal Mori dream surfaces of general type

Abstract: This talk introduces a new approach to study minimal surfaces of general type with $p_g = 0$ via their Cox rings, especially using the notion of combinatorially minimal Mori dream space introduced by Hausen.

First, general properties of combinatorially minimal Mori dream surfaces will be given.

Second, several examples of Mori dream surfaces of general type with $p_g = 0$ will be provided, along with explicit computation of their effective cones.

Then their fibrations and combinatorially minimal models will be discussed.

Many minimal surfaces of general type with $p_g = 0$ arise from the minimal resolutions of combinatorially minimal Mori dream surfaces. A joint work with Kyoung-Seog Lee.

Justin Lacini (Princeton University)

Title: Logarithmic bounds on Fujita's conjecture

Abstract: A longstanding conjecture of T. Fujita asserts that if X is a smooth complex projective variety of dimension n and if L is an ample line bundle, then K_X+mL is basepoint free for m>=n+1. The conjecture is known up to dimension five by the work of Reider, Ein, Lazarsfeld, Kawamata, Ye and Zhu. In higher dimensions, breakthrough work of Angehrn, Siu, Helmke and others showed that the conjecture holds if m is larger than a quadratic function in n. We show that for n>=2 the conjecture holds for m larger than n(loglog(n)+3). This is joint work with L. Ghidelli.

Haiding Liu (Sun Yat-Sen University)

Title: On degrees of canonical Fano threefolds

Abstract: The anti-canonical degree of a Fano threefold is an important invariant of Fano threefolds, playing a key role in the classification of smooth cases. For the classification of canonical Fano threefolds, it is also crucial to understand the behavior of their degrees. It is conjectured that the upper bound of degrees is 72. In this talk, I will present that this conjecture holds when the Fano threefold is Q-factorial and of Picard number one. This is a joint work with Chen Jiang and Jie Liu.

Wenfei Liu (Xiamen University)

Title: On the Iitaka volumes of log canonical surfaces and threefolds

Abstract: For projective log canonical pairs (X, B) with nonnegative Iitaka--Kodaira dimension, the Iitaka volume measures the asymptotic growth of the pluricanonical systems. In the general type case, it is just the usual notion of volume, and plays a key role in the classification theory. In this talk, we are interested in the distribution of the Iitaka volumes for pairs with intermediate Iitaka--Kodaira dimension. It is expected that the set of Iitaka volumes satisfies the descending chain condition (DCC), once the coefficients of the boundary divisor B are contained in a given DCC set. In this talk, built on the work of several previous authors, I will confirm this expectation in dimension at most 3. For surfaces, we go on to find the minimal possible Iitaka volume when the Iitaka-Kodaira dimension is 1. The talk is based on joint work with Guodu Chen and Jingjun Han.



Xin Lü (East China Normal University)

Title: Bounds on the Albanese maps of surfaces of general type

Abstract: Let SS be a smooth irregular projective surface of general type over the complex number, and $a: S \to Alb(S)$ its Albanese map. I will discuss the boundedness problems on the fibers of the Albanese map. Namely, if the image a(S) is of dimension two, what is upper bound on the degree of a? And if the image a(S) is of dimension one, what is the upper bound on the genus of the Albanese fibration induced by a? I will also explain our observation on the relation of these two problems. This is based on joint works with Songbo Ling.

Aleksandr Pukhlikov (University of Liverpool)

Title: Birational geometry of Fano-Mori fibre spaces

Abstract: Starting with the pioneer paper of Iskovskikh and Manin on three-dimensional quartics (1971), the birational rigidity has been shown for quite a lot of classes of Fano varieties in various dimensions. However, for fibrations over a positive-dimensional base the birational rigidity until recently was known only when either the fibre or the base is one-dimensional. In my talk, I will discuss my recent results on the birational rigidity of Fano-Mori fibre spaces with both the fibre and the base of high dimension.

Miles Reid FRS (University of Warwick)

Title: The Semple Cremona transformation of PP^4 (joint work with Alessio Corti)

Abstract: Around 1928 Semple constructed the Cremona transformation phi: PP^4 - -> PP^4 as the birational map given by the 5 Pfaffian quadrics generating the ideal of a genus 1 curve E in PP^4. Its locus of indeterminacy is E, and its critical locus the secant variety Sec^2 E (a quintic hypersurface in PP^4). The map phi contracts Sec^2 E to the symmetric square $S = S^2(E)$ embedded as a quintic scroll S in the image PP^4. As an abstract surface, S is the unique stable PP^1-bundle over E (due to Atiyah). Its ideal I_S in PP^4 is generated by 5 cubic forms G_i, that define the inverse Cremona transformation psi. We offer a number of representations of the transformation PP^4 <- -> PP^4 given by phi and psi as projective closures in bigraded algebra interpretations. One of them, possibly surprisingly, is as a linear section in PP^10 of the spinor Grassmannian OGr(5,10) in PP^15 of our previous paper [CR].

Tong Zhang (East China Normal University)

Title: Moduli spaces of threefolds on the three Noether lines

Abstract: I will talk about the classification of canonical threefolds on the first three Noether lines by describing their moduli spaces, which includes an explicit stratification, an estimate of the number of irreducible components and the dimension formula for every such moduli space. The key idea behind the proof will also be discussed. This is a joint work in progress with S. Coughlan, Y. Hu and R. Pignatelli.



Accommodation

For registered participants who are outside Beijing, your accommodation is assigned to one of the following three places. Breakfast is included no matter which hotel you are in.

Zhongguanyuan Global Village PKU 北京大学中关新园 No.126 Zhongguancun North Street, Haidian District, Beijing 北京市海淀区中关村北大街 126 号

The Lakeview Hotel Beijing 北大博雅国际酒店 No. 127 Zhongguancun North Street, Haidian District, Beijing 北京市海淀区中关村北大街 127 号

Crowne Plaza Beijing Zhongguancun 北京中关村皇冠假日酒店 No. 106, Zhi Chun Road, Haidian District, Beijing 北京市海淀区知春路 106 号

You should have already received an email regarding which hotel you are assigned to. If not, please contact Jihao Liu <u>liujihao@math.pku.edu.cn</u>.

Directions

From Beijing Capital Airport (PEK):

Taxi: About 1-2 hours (depending on the time of your arrival), 100-150 RMB. Metro: Take Capital Airport Express to Sanyuanqiao (三元桥) station, then transfer to subway line 10 (counter-clockwise loop) to Haidian Huangzhuang (海淀黄庄) station, then transfer to subway line 4 (northbound) to Peking University East Gate (北京大学东门) station. About 1.5 hours and 30 RMB.

From Beijing Daxing Airport (PKX):

Taxi: About 1.5-2 hours (depending on the time of your arrival), 300-400 RMB.

Metro1: Take Daxing Airport Express to Caoqiao (草桥) station, then transfer to subway line 19 (northbound) to Mudanyuan (壮丹园) station, then transfer to subway line 10 (counter-clockwise loop) to Haidian Huangzhuang (海淀黄庄) station, then transfer to subway line 4 (northbound) to Peking University East Gate (北京大学东门) station. About 1.5 hours and 40 RMB.

Metro2: Take Daxing Airport Express to Caoqiao (草桥) station, then transfer to subway line 10 (clockwise loop) to Haidian Huangzhuang (海淀黄庄) station, then transfer to subway line 4 (northbound) to Peking University East Gate (北京大学东门) station. About 2 hours and 40 RMB. Metro+Taxi: Take Daxing Airport Express to Caoqiao (草桥) station, then go out and take a taxi there. About 1.5 hours and 120-180 RMB.

From Beijing South Railway Station:

Taxi: About 1 hour, 100 RMB.

Metro: Take subway line 4 (northbound) to Peking University East Gate (北京大学东门) station. About 40 minutes and 5 RMB.



Dining information

Breakfast: served in your hotel daily.

Lunch: Buffet is served in Western Restaurant at Zhongguanxinyuan Global Village building 1 (中关新园 时光西餐厅;中关新园 1 号楼 2 层) during the conference every day (May 28th to June 1st) Dinner: not served during the conference (except the banquet on Tuesday, May 29th). Banquet on May 29th: Guanhu Room at Zhongguanxinyuan Global Village building 1 (中关新园观湖厅; 中关新园 1 号楼 2 层)

Dining inside campus

Most restaurants on campus are only open to students. There are also restaurants/cafés on campus that are open to the public. These include:

Chinese restaurant of Shaoyuan (勺园中餐厅) Western restaurant of Shaoyuan (勺园西餐厅) 3W Coffee Paradise Coffee Faculty's house (教师之家) (<u>Note: you may need to be accompanied by a faculty/postdoc to go there</u>). There are also food trucks on campus.

Dining outside campus

Restaurants are available in your hotel.

Changchunyuan (畅春园) area: there are many restaurants outside the small west gate (小西门).

Haidianhuangzhuang (海淀黄庄) area, Wudaokou (五道口) area, and Suzhoujie (苏州街) area have many fancier places and some shopping malls. About 30 minutes walking distance from the campus.

For foreign visitors who want to try some of the best dishes/pubs in Beijing, we recommend visiting Sanlitun (三里屯) area or the Nanluoguxiang (南锣鼓巷) area. You need to take public transport/taxi to these places.



Entering the campus

For all participants, please bring your ID card/passport/合胞证 when entering the campus. If you have any trouble entering the campus, please contact <u>liujihao@math.pku.edu.cn</u> immediately.

For participants assigned in The Lakeview Hotel Beijing (北大博雅国际酒店)

We will make appointments for you entering and exit the campus through the small east gate (小东门) and the southeast gate (东南门) everyday, from **7 a.m. to 7 p.m.**

For participants assigned in all other places

We will make appointment for you to enter and exit the campus through the southeast gate (东南门) every day, from **7 a.m. to 7 p.m.**



Maps

On the map of the campus:

Red Star= Zhihua building (where the conference will be held). Note: please enter through the **west gate of Zhihua building** (all other gates require face scanning).

Blue circle=Gates for which we shall reserve entrance for you. You can exit through any gate, but only enter through the southeast gate/small east gate.

Purple star=Hotels. Zhongguanxinyuan is also the place where lunch/banquet is going to be served.

Black line=How you go from your hotel to Zhihua Building.

Pink line=How you go from the subway station to your hotel.

Green square=Restaurants/café where you can pay by cash/credit card/Alipay/Wechat pay There are also restaurants in your hotel. Some names of the restaurants/café are provided. You may need a faculty/postdoc to accompany you to enter the faculty's house.

Red triangle= Math Center (BICMR)

Light blue rectangle= grocery store

On the subway map:

Red Star= Peking University East gate station

Blue triangles = Airports/Train station

北京大学校园图 Peking University Map







