

《利息理论与应用》第七章重点题答案

1.

$$1) A = x \cdot \frac{1.08^{10}}{1.05^{10}}$$

$$2) B = x \cdot \left(\frac{1.08}{1.05}\right)^{10}$$

$$\Rightarrow A : B = 1 : 1$$

6.

$$\begin{aligned} & 80a_{\overline{10}|2\%} + 1000 \times 98\%q^{10} \\ &= \frac{8000}{12} + \left(980 - \frac{8000}{12}\right) \cdot 1.12^{-10} \\ &= 767.5516 \end{aligned}$$

7.

$$p(I=1) = 90\% = 1 - p(I=0)$$

$$EI = 0.9, V_{ar}(I) = p \cdot q = 0.09, s = 0.3$$

$$x = 1000q \cdot I$$

$$1) Ex = 1000 \cdot q \cdot EI = \frac{1000}{1.25} \cdot 0.9 = 720$$

$$2) V_{ar}(x) = 1000^2 q^2 V_{ar}(I) \cdot s(x) = 1000qs = \frac{1000}{1.25} \cdot 0.3 = 240$$

$$3) \left(\frac{1000}{720} - 1\right) - 25\% = 13.89\%$$

10.

$$\begin{aligned} Ex_1 &= R \cdot q \cdot 0.99 + \dots + Rq^{15} \cdot 0.99^{15} \\ &= R \cdot \frac{1 - \left(\frac{0.99}{1.12}\right)^{15}}{1 - \left(\frac{0.99}{1.12}\right)} \cdot \frac{0.99}{1.12} = 1.5 \times 10^5 \end{aligned}$$

$$Ex_2 = R \cdot \frac{1 - \left(\frac{0.98}{1.12}\right)^{15}}{1 - \left(\frac{0.98}{1.12}\right)} \cdot \frac{0.98}{1.12}$$

$$Ex_3 = R \cdot \frac{1 - \left(\frac{0.98}{1.14}\right)^{15}}{1 - \left(\frac{0.98}{1.14}\right)} \cdot \frac{0.98}{1.14}$$

$$\Rightarrow Ex_2 = 141500, Ex_3 = 128300$$

15.

$$\begin{aligned} 6 \cdot \sum q_r^t + 100(1+r)^{-6} &= 6a_{\overline{6}|2\%} + 100 \cdot 1.12^{-6} \\ 10 \cdot \sum q_r^t + 100(1+r)^{-6} &= 10a_{\overline{6}|8\%} + 100 \cdot 1.08^{-6} \\ 200(1+r)^{-6} &= 30(a_{\overline{6}|2\%} - a_{\overline{6}|8\%}) + 2.5 \cdot 1.12^{-6} \\ \Rightarrow (1+r)^{-6} &= -0.625 + 1.25 \cdot 1.12^{-6} + 0.375 \cdot 1.08^{-6} \\ &= 0.24460251 \\ \Rightarrow r &= 26.45\% \end{aligned}$$

27.

$$p(i) = 35000 \times 1.08^5 \times (1+i)^{-5} + 50000 \times \frac{0.08}{i} - 85000 \times 1.08^{10} \times (1+i)^{-10}$$

$$p(8\%) = 0$$

$$p'(i) = -5 \times 35000 \times 1.08^5 (1+i)^{-6} - 50000 \times 0.08 \cdot i^{-2} + 10 \times 85000 \times 1.08^{10} (1+i)^{-11}$$

$$p'(8\%) = -5 \times 35000 \times \frac{1}{1.08} - \frac{50000}{0.08} + \frac{10 \times 85000}{1.08} = 0$$

$$p''(i) = 30 \times 35000 \times 1.08^5 \times (1+i)^{-7} + 2 \times 50000 \times 0.08 \cdot i^{-3} - 11 \times 10 \times 85000 \times 1.08^{10} (1+i)^{-12}$$

$$p''(8\%) > 0$$

∴ 该方案是最优的投资方案

28.

$$PVL(i) = 10^4 a_{\overline{3}|i}, PVL(10\%) = 3790g$$

$$PVL'(i) = 10^4 \left[-\frac{1}{(1+i)^2} - \frac{2}{(1+i)^3} - \dots - \frac{5}{(1+i)^6} \right] = -10^4 \times \frac{1}{1+i} Ia_{\overline{3}|i}$$

$$PVL''(i) = 10^4 \left[\frac{2}{(1+i)^3} + \frac{6}{(1+i)^4} + \frac{12}{(1+i)^5} + \frac{20}{(1+i)^6} + \frac{30}{(1+i)^7} \right]$$

设三种债券持有额分别为 X, Y, Z

$$PVA(i) = X(1+10\%)(1+i)^{-1} + Y(1+10\%)^3(1+i)^{-3} + Z(1+10\%)^5(1+i)^{-5}$$

$$PVA(10\%) = X + Y + Z$$

$$PVA'(i) = -X1.1(1+i)^{-2} - 3Y1.1^3(1+i)^{-4} - 5Z1.1^5(1+i)^{-6}$$

$$PVA'(10\%) = -\frac{X}{1.1} - \frac{3Y}{1.1} - \frac{5Z}{1.1}$$

$$PVA''(i) = 2X \cdot 1.1(1+i)^{-3} + 12Y1.1^3(1+i)^{-5} + 30Z1.1^5(1+i)^{-7}$$

$$PVA''(10\%) = \frac{2X}{1.1^2} + \frac{12Y}{1.1^2} + \frac{30Z}{1.1^2}$$

由免疫技术应用：

$$PCVL(10\%) = PVA(10\%)$$

$$PVL'(10\%) = PVA'(10\%)$$

$$PCL''(10\%) < PVA''(10\%)$$

$$\Rightarrow \begin{cases} X + Y + Z = 37908 & (1) \\ -\frac{X}{1.1} - \frac{3Y}{1.1} - \frac{5Z}{1.1} = -10^4 \cdot \frac{1}{1.1} Ia_{\overline{5}|10\%} & (2) \\ PVA''(10\%) > PVL''(10\%) & (3) \end{cases}$$

当 $X = Y = Z = 12636$ 时, $X + 3Y + 5Z = 113724 \neq 106526$

∴ 三种债券的投资额均为 12636 元不是最优
如果取 $Z=9624$ 则可以由 q 和 (1) 和 (2) 得到

$$X = 13223, Y = 15061$$