单参数指数族.

- 总体密度为$f(x,\theta)=S(\theta)h(x)\exp\{C(\theta)T(x)\}$, $C(\theta)$ 严格增.

  $H_0:\theta\leqslant(\geqslant)\theta_0$. $\mathcal{W}=\{\vec{x}:\sum_{i=1}^n T(x_i)>(<)c\}$.

二、$X\sim N(\mu,\sigma^2)$，均值μ的检验.

- (1) σ^2 已知: $T(\vec{x})=\frac{\sqrt{n}(\bar{x}-\mu_0)}{\sigma}$.

  $H_0:\mu=(\leqslant,\geqslant)\mu_0$. $\mathcal{W}=\{\vec{x}:|T(\vec{x})|>c\}$, $P(|Z|>c)=\alpha$.

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- (2) σ^2 未知: $T(\vec{x})=\frac{\sqrt{n}(\bar{x}-\mu_0)}{\sqrt{\frac{1}{n-1}\sum_{i=1}^n(x_i-\bar{x})^2}}$.

  $H_0:\mu=(\leqslant,\geqslant)\mu_0$.

  $\mathcal{W}=\{\vec{x}:|T(\vec{x})|>c\}$, $P(|T_{n-1}|>c)=\alpha$.

  $\mathcal{W}=\{\vec{x}:T(\vec{x})(>,<-)c\}$, $P(T_{n-1}>(<-)c))=\alpha$.

三、正态总体$X \sim N(\mu, \sigma^2)$，方差σ^2 的检验

(1) 双边问题　$H_0 : \sigma^2 = \sigma_0^2 \leftrightarrow H_1 : \sigma^2 \neq \sigma_0^2$.

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  $\hat{\sigma}^2 = \underline{\frac{1}{n}\sum_{i=1}^n (x_i - \hat{\mu})^2}$, $\hat{\sigma}_0^2 = \sigma_0^2$.
  若$\sigma^2 = \sigma_0^2$, 则$T(\vec{X}) := \frac{n\hat{\sigma}^2}{\sigma_0^2} \sim \chi^2(n)$ 或$\chi^2(n-1)$.

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- 似然比: $\lambda(\vec{x}) = \frac{L(\hat{\theta})}{L(\hat{\theta}_0)} = \left(\frac{\hat{\sigma}^2}{\sigma_0^2}\right)^{-\frac{n}{2}} \exp\{\frac{n\hat{\sigma}^2}{2\sigma_0^2}\}e^{-\frac{n}{2}}$.
  $L(\hat{\theta}) = (\frac{1}{\sqrt{2\pi}\hat{\sigma}})^n \exp\{-\frac{1}{2\hat{\sigma}^2}\underline{\sum_{i=1}^n (x_i - \hat{\mu})^2}\} = (2\pi\hat{\sigma}^2)^{-\frac{n}{2}} e^{-\frac{n}{2}}$.
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- 改用$\tilde{c}_1, \tilde{c}_2$, $P(T < \tilde{c}_1) = P(T > \tilde{c}_2) = \frac{\alpha}{2}$. $T = \chi_n^2$ 或χ_{n-1}^2

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$$\lambda(\vec{x}) = \begin{cases} 1, & \sigma_0^2 < \hat{\sigma}^2 (\Leftrightarrow T(\vec{X}) > n); \\ \left(\frac{T(\vec{x})}{n}\right)^{-\frac{n}{2}} e^{\frac{T(\vec{x})}{2}} e^{-\frac{n}{2}}, & \sigma_0^2 \geqslant \hat{\sigma}^2 (\Leftrightarrow T(\vec{X}) \leqslant n). \end{cases}$$

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- 一般不用担心σ^2 小, 所以不必处理$H_0 : \sigma^2 \leqslant \sigma_0^2$.

四、两正态$X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$ 的参数检验

数据: $\vec{x} = (x_1, \cdots, x_{n_1})$, $\vec{y} = (y_1, \cdots, y_{n_2})$.

- $H_0 : \mu_1 = \mu_2 \leftrightarrow H_1 : \mu_1 \neq \mu_2$

  (1) σ_1^2, σ_2^2 已知;

  (2) σ_1^2, σ_2^2 未知, 但知道$\sigma_1^2 = \sigma_2^2$;

  (3) σ_1^2, σ_2^2 未知. (太过复杂, 略)

- $H_0 : \sigma_1^2 = \sigma_2^2 \leftrightarrow H_1 : \sigma_1^2 \neq \sigma_2^2$

  (4) μ_1, μ_2 已知. (自习, 略).

  (5) μ_1, μ_2 未知.

- 单边假设检验问题类似.

(1) σ_1^2, σ_2^2 已知; 检验$H_0: \mu_1 = \mu_2 \leftrightarrow H_1: \mu_1 \neq \mu_2$..

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- 否定域: $\mathcal{W} = \{(\vec{x}, \vec{y}) : |\bar{x} - \bar{y}| > c\}$.
- 根据水平α 定c:
  $P_{H_0}((\vec{X}, \vec{Y}) \in \mathcal{W}) = P(|Z| > c/\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}) = \alpha.$

(2) σ_1^2, σ_2^2 未知, 但知道$\sigma_1^2 = \sigma_2^2 (= \sigma^2)$.

检验$H_0 : \mu_1 = \mu_2 \leftrightarrow H_1 : \mu_1 \neq \mu_2$.

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- 注：上面的否定域不是广义似然比否定域；广义似然比否定域也不是UMPU, 但能导出否定域的如上形式; 进一步研究知存在UMPU.

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- $\frac{v_i - np_i}{\sqrt{np_i}} \overset{1<<m \text{故} p_i<<1}{\approx} \frac{v_i - np_i}{\sqrt{np_i(1-p_i)}} \overset{m<<n, CLT}{\sim} N(0,1)$.

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$$F_n(x) = \begin{cases} 0, & x < X_{(1)}, \\ k/n, & X_{(k)} \leqslant x < X_{(k+1)}, k = 1, \cdots, n-1, \\ 1, & x \geqslant X_{(n)}. \end{cases}$$

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- 将数据$x_1, \cdots, x_n$ 排序得到$x_{(1)} < \cdots < x_{(n)}$，则$D_n(\vec{x}) = \max_{1 \leqslant k \leqslant n} \max\left\{\left|\frac{k}{n} - F_0(x_{(k)})\right|, \left|F_0(x_{(k)}) - \frac{k-1}{n}\right|\right\}$.
- 否定域: $\mathcal{W} = \{\vec{x} : D_n(\vec{x}) > c\}$.
- 根据α 定c. $P(D_n(\vec{X}) > c) = \alpha$.

推广: $\mathcal{F}_0 = \{F(x,\theta), \theta \in \Theta\}$

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- 注：H_0假设下D_n的极限分布是否存在？以上检验只是近似检验.