摘要：Statistical physicists and probabilists often try to understand the macroscopic behavior of systems consisting of many microscopic random inputs, which can give rise to interfaces between two phases at a critical temperature. This can be modeled via the scaling limit behavior (macroscopic behavior) of discrete lattice models (microscopic inputs). In most cases, the limits of these discrete models become deterministic (in the spirit of Law of Large Number); and in some critical cases, the limits can remain random, which is of particular interest.

Oded Schramm's SLE (Stochastic Loewner Evolution) processes have led mathematicians and physicists to a clean and novel understanding of the scaling limits of discrete models in two dimensions. A chordal SLE is a random non-self-traversing curve in a simply connected domain, joining two prescribed boundary points of the domain. And it is the only one-parameter family (usually indexed by a positive real number κ) of random planar curves that satisfies conformal invariance and domain Markov property.

CLE (Conformal Loop Ensemble) is the limit geometric object when one tries to consider the ``entire'' scaling limit of discrete model (in contrast with only one interface which turns out to be the SLE process). A simple CLE can be viewed as a random countable collection of disjoint simple loops in the unit disk that are non-nested. It is the only one-parameter family that satisfies conformal invariance and (loops-configuration's) domain Markov property. As one can somehow expect, each loop in CLE is a loop whose geometry is a SLE-type loop, with the same parameter κ.

The GFF (Gaussian Free Field) is a natural two-dimensional time analog of Brownian motion, that has been used extensively as a basic building block in Quantum Field Theories. Like Brownian motion, it is a simple random object of widespread application and great intrinsic beauty. It plays an important role in statistical physics, the theory of random surfaces, and quantum field theory. The geometry of the two-dimensional Gaussian Free Field, i.e. the fact that it was possible to describe geometric lines in this very irregular distribution, has been discovered recently, and let to a number of recent developments.

SLE, CLE and GFF are three important related planar random structures and the present thesis will explore aspects of these three objects and of the relation between them. In this talk, we will discuss the relations between these three objects.