The Z-invariant massive Laplacian on isoradial graphs

Cédric Boutillier

Université Pierre et Marie Curie

After having explained the notion of Z-invariance for models of statistical mechanics, we introduce a one-parameter family (depending on the elliptic modulus k) of Z-invariant massive Laplacians defined on isoradial graphs. We prove an explicit formula for the inverse operator, the massive Green function, which has the remarkable property of only depending on the local geometry of the graph. We explain consequences of this result for the model of spanning forests, in particular the proof of an order 2 phase transition at $k = 0$ with the model of critical spanning trees introduced by Kenyon. Finally, we consider the spectral curve of this massive Laplacian and prove that it is a Harnack curve of genus 1. This is joint work with Béatrice de Tilière and Kilian Raschel.

Capacity of the range of simple random walks on Z^d

Yinshan Chang

Sichuan University

The capacity measures the size of the range of a simple random walk from the point of view of another independent SRW. We prove a weak law of large numbers for the capacity of the range of SRWs on Z^4. On Z^3, there is no such law of large numbers. The capacity, properly scaled, converges in distribution towards the corresponding quantity for three dimensional Brownian motion.

Large deviation for empirical distribution of branching random walk

Xinxin Chen

Université Claude Bernard Lyon I

Given a super-critical branching random walk on $\mathbb{R}$ started from the origin, let $Z_n(\cdot)$ be the counting measure which counts the number of individuals at the n-th generation located in a given set. Under some mild conditions, it is known in Biggins (1990) that for any interval $A \subset \mathbb{R}$, $\frac{Z_n(\sqrt{n}A)}{Z_n(\mathbb{R})}$ converges a.s. to $\nu(A)$, where ν is the standard Gaussian measure. We investigate the large deviation of this convergence in Schröder case and in Böttcher case. This is joint work with Hui He (Beijing Normal University).

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Gaussian bounds and collisions of random walks on weighted lattices

Xinxing Chen

Shanghai Jiaotong University

We consider a weighted lattice $\mathbb{Z}^d$ with conductance $\mu_e = |e|^{-\alpha}$. We show that the heat kernel of a variable speed random walk on it satisfies a two-sided Gaussian bound by using an intrinsic metric. We also show that when $d = 2$ and $\alpha \in (-1, 0)$, two independent random walks on such weighted lattice will collide infinite many times while they are transient.

Covering the two-dimensional torus and random interlacement

Francis Comets

Université Paris Diderot

The cover time is the time needed for the simple random walk to visit all points on the lattice, and in the continuum, for the Wiener sausage of radius 1 to cover the torus of linear size n. As a maximum of correlated random variables (here, with logarithmic decay) it has interesting asymptotics. In higher dimension random interlacements, introduced by Sznitman to describe the local covering picture with a fixed intensity, still give a reliable account at large densities up to cover time. In dimension 2, it can be used as adescription of the neighborhood of an unvisited site, provided that the paths used in the interlacement are random walk and Brownian motion conditioned not to visit a ball.

The Z-invariant Ising model via dimers

Béatrice de Tilière

Université Paris-Est

Introduced by Baxter, the Z-invariant Ising model is a 2-dimensional Ising model defined on isoradial graphs. The coupling constants satisfy the Yang-Baxter equations; they are written in terms of elliptic functions depending on an elliptic parameter k, where k parametrizes the external temperature. For the specific value $k = 0$, the Ising model is critical. Fisher's correspondence allows to study the Ising model through an associated dimer model, and this is the approach we follow. We prove an explicit formula, only depending on the local geometry of the graph, for dimer probabilities. We prove an explicit and also local expression for the free energy of the model. We also prove an order 2 phase transition for the Ising model at $k = 0$, and show that it is the same as that of the spanning forests model of the previous talk. This is joint work with Cédric Boutillier and Kilian Raschel.

Scaling limit of inhomogeneous random graphs

Thomas Duquesne

Université Pierre et Marie Curie

Joint work with Nicolas Broutin and Minmin Wang.

We consider inhomogeneous random graphs that extend Erdos-Renyi case; these models have been studied first by Aldous & Limic [2, 3] and their connected components evolve as a multiplicative coalescent: namely, let N be the number of vertices and let $w_1, \ldots, w_N$ be a set of positive weights; we independently put an edge between vertices i and j with probability $p_{i,j} = 1 - e^{-w_i w_j / s}$ (in the case we consider, $s = w_1 + \ldots + w_N$).

Our results are the following: we first generate such graphs by an exploration governed by a LIFO queue. This point of view allows to code an appropriate spanning tree of the graph thanks to a contour process (and a modified Lukasiewicz path) and to get a simple control on the surplus edges. This is reminiscent of the Addario-Berry, Broutin & Goldschmidt [1], but the technical details are quite distinct. This coding allows to embed such graphs into Galton-Watson tree: this new idea helps to identify and define the the possible rescaled limits of such graphs. Namely, based on this representation, we prove that, under the assumption that the degree distribution of a size-biased vertex is in the domain of attraction of an infinite divisible law, the connected components of the graphs, after a suitable scaling in the graph metric, converge in distribution towards a collection of compact metric spaces with respect to the Gromov-Hausdorff-Prokhorov topology (actually, we prove this result via the contour function of the spanning trees of the graphs). We give a direct construction of the limit metric spaces thanks to a special process Y that can be informally described as a spectrally positive Lévy process whose repeated jumps have been removed. This process already appeared in Aldous & Limic [3]. The limiting continuum graph is called the *Continuum Multiplicative Graph*. It can be embedded in Lévy trees: this embedding allows to export easily properties of Lévy trees to Continuum Multiplicative Graph such as Hausdorff and packing dimensions.

Thanks to Janson's result in [5], this limit-theorem applies to more general inhomogeneous $p_{i,j}$. This limit theorem extends the important case of Erdos-Renyi graphs (i.e. the cases where the w_i are constant) that have been investigated by Addario-Berry, Broutin & Goldschmidt [1]. It also significantly extends the cases considered by Bhamidi, van der Hofstad & Sanchayan [4] who (roughly speaking) considered cases where w_j are close to j^{-a} by use of quite distinct techniques.

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Probabilistic representation formula for Navier-Stokes equations on manifolds

Shizan Fang

Université de Bourgogne

The well-known Constantin-Iyer's representation formula for NS equations is extended to manifolds. In this talk, we will explain how the non-parallelism of manifolds will produce difficulties and to handle them.

Multivariate analogs of classical discrete distributions and their properties

Yury Khokhlov

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Department of Mathematical Statistics

The role of some discrete distributions such as Bernoulli, binomial, geometric, negative binomial, Poisson, Polya-Aeppli and others in applications of probability theory and mathematical statistics is well known. We propose some variant of multivariate distribution whose components have given univariate discrete distributions. In fact we consider some very general variant of so called reduction method. We find the explicit form of mass function and generating function of such distribution and investigate its properties. We prove that our construction is unique in natural exponential families of distributions. Our results are the generalization and unification of many results of other authors.

Representation of asymptotic values for nonexpansive stochastic control systems

Juan Li

Shandong University

We will study the limit behavior of the Abel mean $\lambda V_\lambda(x)$ which is introduced by BSDE as the discount factor λ tends to zero. We show that the family of functions $\{\lambda V_\lambda(x)\}_\lambda$ is equicontinuous and equibounded under appropriate conditions. And we will characterize the limit value of $\lambda V_\lambda(x)$.

It's based on a joint work with Nana Zhao (SDU, Weihai).

Energy saving approximation of random processes

M.A. Lifshits

St.Petersburg State University

The classical linear prediction problem for a a wide sense stationary process consists of finding an element in the linear span of the past values providing the best possible mean square approximation to the current and future values of the process. In this talk we investigate this and some other similar problems where, in addition to prediction quality, optimization takes into account other features of the objects we search for. One of the most motivating examples of this kind is an approximation of a stationary process by a stationary differentiable process taking into account the kinetic energy that the latter spends in its approximation efforts. We also provide appropriate extensions of the classical Kolmogorov and Krein prediction singularity criteria and Kolmogorov's criterion of error-free interpolation.

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Harmonic measure for biased random walk on a Galton–Watson tree

Shen Lin

Université Pierre et Marie Curie

Consider random walk λ-biased towards the root on a leafless Galton–Watson tree, whose offspring distribution is non-degenerate and has finite mean $m > 1$. In the transient regime $0 < \lambda < m$, the loop-erased trajectory of the biased random walk defines the λ-harmonic ray, whose law is called the λ-harmonic measure on the boundary of the Galton–Watson tree. In this talk, I will present some recent results concerning the Hausdorff dimension of the λ-harmonic measure and the average number of children of the vertices visited by the λ-harmonic ray.

Asymptotic expansions for the distribution of particles in a branching random walk

Quansheng Liu

Université Bretagne-Sud

Consider a branching random walk in which each particle gives birth to children according to a fixed offspring distribution, and each child moves on the real line according to a fixed law. For the normalized counting measure which counts the number of particles of generation n in a given region, we show asymptotic expansions in the central limit theorem under rather weak assumptions on the moments of the branching and moving laws. In the proofs, we use the Edgeworth expansion in the central limit theorem for sums of independent random variables, truncating techniques and martingale approximations. (Joint work with Gao Zhiqiang of Beijing Normal University.)

Quasi-linear (stochastic) PDEs with time-fractional derivatives

Wei Liu

Jiangsu Normal University

In this talk, we will first recall the classical and generalized variational framework for stochastic PDEs, then we will introduce a method to solve (stochastic) evolution equations with time-fractional derivative based on monotonicity techniques. Applications include deterministic and stochastic quasi-linear partial differential equations with time-fractional derivatives, including time-fractional (stochastic) porous media equations (including the case where the Laplace operator is also fractional) and p-Laplace equations as special cases.

A characterization of quadratic transportation-information inequalities

Yuan Liu

AMSS, Chinese Academy of Sciences

We give a new characterization of quadratic transportation-information inequalities via certain Lyapunov condition. We also obtain the stability of these inequalities under bounded perturbations.

Non-thin local sets of the 2D GFF and Minkowski content

Titus Lupu

Université Pierre et Marie Curie

We consider local sets of the continuum Gaussian Free Field in dimension 2. These are the sets along which the GFF admits a Markovian decomposition. Non-thin local sets are those on which the field is non-trivial, i.e. are "charged" by the field. An example of these are the first passage sets. We show that for a large class of such non-thin local sets, the value of the GFF on the set coincides with the Minkowski content of the set in the gauge $|\log r|^{1/2} r^2$. Further, we show that the GFF can be constructed out of a Brownian loop-soup, by taking the Minkowski content of clusters of Brownian loops. The last property is related to Dynkin's isomorphism.

This is a joint work with Juhan Aru and Avelio Sepulveda at ETH Zurich.

Distances between random orthogonal matrices and independent normals

Yutao Ma

Beijing Normal University

Let Γ_n be an $n \times n$ Haar-invariant orthogonal matrix. Let Z_n be the $p \times q$ upper-left submatrix of Γ_n, where $p = p_n$ and $q = q_n$ are two positive integers. Let G_n be a $p \times q$ matrix whose pq entries are independent standard normals. In this paper we consider the distance between $\sqrt{n}Z_n$ and G_n in terms of the total variation distance, the Kullback-Leibler distance, the Hellinger distance and the Euclidean distance. We prove that each of the first three distances goes to zero as long as pq/n goes to zero, and not so this rate is sharp in the sense that each distance does not go to zero if (p, q) sits on the curve $pq = \sigma n$, where σ is a constant. However, it is different for the Euclidean distance, which goes to zero provided pq^2/n goes to zero, and not so if (p, q) sits on the curve $pq^2 = \sigma n$. A previous work by Jiang (2006) shows that the total variation distance goes to zero if both $p/\sqrt{n}$ and $q/\sqrt{n}$ go to zero, and it is not true provided $p = c\sqrt{n}$ and $q = d\sqrt{n}$ with c and d being constants. One of the above results confirms a conjecture that the total variation distance goes to zero as long as $pq/n \to 0$ and the distance does not go to zero if $pq = \sigma n$ for some constant σ.

A study of percolation on random triangulations via generating functions

Grégory Miermont

École Normale Supérieure de Lyon

We study site and bond percolation on random planar triangulations. Using a recursive decomposition of maps with a monochromatic boundary, and counting methods using generating functions whose origin can be traced back to Tutte, we identify the critical point for these models, as well as information on the size and the geometry of the clusters in all of the usual regimes (subcritical, critical and supercritical). This is joint work with Olivier Bernardi and Nicolas Curien.

Integro-local limit theorems for multidimensional compound renewal processes

A.A.Mogulskii

Sobolev Institute of Mathematics of the Russian Academy of Sciences
and Novosibirsk State University

Let $(\tau, \boldsymbol{\zeta}), (\tau_1, \boldsymbol{\zeta}_1), (\tau_2, \boldsymbol{\zeta}_2), \cdots$ be a sequence of i.i.d. random vectors,

$$\tau > 0, \quad \boldsymbol{\zeta} := (\zeta_{(1)}, \cdots, \zeta_{(d)}) \in \mathbf{R}^d, \quad d \geq 1;$$

$$T_0 := 0, \;\; T_n := \sum_{j=1}^{n} \tau_j, \quad Z_0 := \mathbf{0}, \;\; \mathbf{Z}_n := \sum_{j=1}^{n} \boldsymbol{\zeta}_j;$$

$$\nu(t) := \max\{k \geq 0 : \; T_k < t\}, \quad \eta(t) := \min\{k \geq 0 : \; T_k \geq t\}.$$

The compound renewal processes (CRPs) $\mathbf{Z}(t), \mathbf{Y}(t)$ for the sequence $(\tau_j, \boldsymbol{\zeta}_j), \; j \geq 1$, are defined as (see [1],[2])

$$\mathbf{Z}(t) := \mathbf{Z}_{\nu(t)}, \quad \mathbf{Y}(t) := \mathbf{Z}_{\eta(t)}, \quad t \geq 0.$$

Let the Cramér moment condition for $(\tau, \boldsymbol{\zeta})$ be met. For a vector $\boldsymbol{x} = (x_{(1)}, \cdots, x_{(d)}) \in \mathbf{R}^d$ put

$$\Delta[\boldsymbol{x}) := [x_{(1)}, x_{(1)} + \Delta) \times [x_{(2)}, x_{(2)} + \Delta) \times \cdots \times [x_{(d)}, x_{(d)} + \Delta), \quad \Delta > 0.$$

We establish the fine asymptotics of the probabilities (integro-local limit theorems for CRPs $\mathbf{Z}(t)$, $\mathbf{Y}(t)$)

$$\mathbf{P}(\mathbf{Z}(t) \in \Delta[\boldsymbol{x})) = ?, \quad \mathbf{P}(\mathbf{Y}(t) \in \Delta[\boldsymbol{x})) = ?$$

in the range of normal and large deviations.

It is a joint work with E.I.Prokopenko.

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On local laws for non-hermitian random matrices and their products

A. Naumov

Skolkovo Institute of Science and Technology

We prove the local circular law for non-Hermitian random matrices and at its generalisation to the products of independent non-Hermitian matrices under weak moment conditions. Our results generalise recent results of P. Bourgade, H.-T. Yau, J. Yin and Y. Nemish. We apply Stein's method and some new ideas which help to simplify the proof of the local laws. The talk will be based on joint results with F. Goetze and A. Tikhomirov.

The comparison principles for L_2-small ball asymptotics for Gaussian processes

Alexander Nazarov

St.Petersburg Department of Steklov Mathematical Institute
and St.Petersburg State University

We discuss the celebrated Wenbo Li comparison principle [1] for the small ball probabilities, its elaboration and applications in last 25 years.

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G-framework of nonlinear expectation theory

Shige Peng

Shandong University

We present recent progresses in nonlinear expectation theory and its applications.

Reinforced random walks, random Schrödinger operators and hitting times of Brownian motions

Christophe Sabot

Université Claude Bernard Lyon I

In the continuation of Pierre Tarrès' talk, we will review recent results on the relation between some self-interacting processes (linearly edge reinforced random walks, vertex reinforced random walks) and random Schrdinger operators. In particular, we will explain how the existence of a delocalized ground state of the Schrödinger operator is related to transience and diffusive behavior of the process. We will also give a brief overview of new relations with hitting times of a family of correlated Brownian motions, generalizing the classical relation between 1-D drifted Brownian motion and the inverse Gaussian law. (Based on joint works with Pierre Tarrès, Xiaolin Zeng.)

The square bias transformation: some properties and applications

Irina Shevtsova

Moscow State University

We give an alternative definition of the square bias transformation in terms of characteristic functions coming back to (Lukacs, 1970) and investigate its properties. In particular, an optimal moment-type estimate of the Kantorovich distance between an arbitrary zero-mean distribution with finite third-order moment and its square bias transformation is presented. The latter yields some new and optimal moment-type bounds on characteristic functions.

Hammersley's trees

Arvind Singh

Université Paris-Sud

(One of) Ulam's problem consists in estimating the length of the longest increasing subsequence in a random permutation. In 1972, Hammersley introduced the so-called "Hammersley's line process" to answer this question. In this talk, I will present a generalization of this particle system called "Hammersley's tree process" which counts, instead, the number of heaps required to store a random permutation. I will also explain why this process gives a natural construction of a stationary random tree embedded in the upper half plane with a prescribed offspring distribution.

Reinforced random walks and statistical physics

Pierre Tarrès

Université Paris Dauphine

We will review recent results on two processes, namely the Edge-reinforced random walk (ERRW), introduced by Coppersmith and Diaconis in 1986, and the Vertex-reinforced jump process (VRJP), proposed by Werner and introduced by Davis and Volkov in 2002.

We will explain how the ERRW and VRJP are explicitly related to several models in statistical physics, namely the supersymmetric hyperbolic sigma model, the random Schrdinger operator and Dynkin's isomorphism.

These correspondences enable us to show recurrence/transience results on the Edge-reinforced random walk, using results of Disertori, Spencer and Zirnbauer (2010) and they also allow us to provide insight into these models. This work is joint with Christophe Sabot, and part of it is also in collaboration with Margherita Disertori, Titus Lupu and Xiaolin Zeng.

Stein's method and characteristic functions

A. Tikhomirov

Institute of Physics and Mathematics
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and Syktyvkar State University, Russia

We consider a method to obtain limit theorems for various functions of independent and weakly dependent random variables, based on characterization of corresponding limit distributions by means of differential equations for their characteristic functions. This approach was first realized by the author in 1976 in his proof of an estimate for the rate of convergence in the Central Limit Theorem for the sum of weakly dependent random variables, [2], [3], being in its turn motivated by the famous Stein paper of 1972, [1]. In the talk we discuss applications of our approach to the Central Limit Theorem for different models including quadratic forms and linear statistics of eigenvalues of random matrices ([4]). We also consider some non-central limit theorems.

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On confidence sets for spectral projectors of covariance matrix

V. Ulyanov

Moscow State University

We offer a bootstrap procedure for building sharp confidence sets for the projectors of a covariance matrix from the given sample. This procedure could be applied for small or moderate sample size and large dimension of observations. The main result states the validity of the proposed procedure for finite samples with an explicit error bound of bootstrap approximation. This bound involves some new sharp results on Gaussian comparison and Gaussian anti-concentration in high dimensions. These are the joint results with V.Spokoiny (WIAS, Berlin) and A.Naumov (Skoltech, Moscow).

Evolutionary jumps described by decomposable branching processes

V.Vatutin

Steklov Mathematical Institute, Moscow, Russia

We consider two models of two-type Galton-Watson processes in which particles of the first type may produce individuals of both types while particles of the second type produce individuals of the second type only.

In the first model both types evolve in a constant environment while in the second model particles of the first type evolve in a random environment while particles of the second type evolve in a constant environment. Assuming that the reproduction laws of particles obey the so-called criticality property we calculate the probability of survival of both processes, prove Yaglom-type conditional limit theorems for the distribution of the population sizes of particles of both types and evaluate the distribution of the distance to the most recent common ancestor of all individuals existing in the population at a distant moment n. It occurs that there is a drastic and unexpected differences in the limiting behavior of the processes.

The results of the talk are partially based on papers [1]-[4].

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Convex hulls of random walks

Vladislav Vysotsky

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and
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I will explain how to compute the expected number of faces in the convex hull of a multidimensional random walk. Remarkably, this quantity does not depend on the distribution of increments of the random walk if this distribution satisfies a certain non-degeneracy assumption.

Elliptic Harnack inequalities for symmetric non-local Dirichlet forms

Jian Wang

Fujian Normal University

In this talk, we present relations and characterizations of various elliptic Harnack inequalities for symmetric non-local Dirichlet forms on metric measure spaces. We allow the scaling function be state-dependent and the state space possibly disconnected. Stability of elliptic Harnack inequalities is established under certain regularity conditions and implication for a priori Hölder regularity of harmonic functions is explored. This talk is based on a joint work with Zhen-Qing Chen and Takashi Kumagai.

Hypergeometric SLE and convergence of critical planar Ising interfaces

Hao Wu

Tsinghua University

Conformal invariance and critical phenomena in two-dimensional statistical physics have been active areas of research in the last few decades. This talk concerns conformally invariant random curves that should describe scaling limits of interfaces in critical lattice models.

The scaling limit of the interface in critical planar lattice model with Dobrushin boundary conditions (b.c.), if exists, should satisfy conformal invariance (CI) and domain Markov property (DMP). In 1999, O. Schramm introduced SLE process, and this is the only one-parameter family of random curves with CI and DMP. In 2010, D. Chelkak and S. Smirnov proved that the interface of critical Ising model on the square lattice does converge to SLE(3). In this talk, we discuss the scaling limit of the pair of interfaces in rectangle with alternating b.c. The scaling limit of the pair of interfaces, if exists, should satisfy CI, DMP and symmetry (SYM). It turns out there is a two-parameter family of random curves satisfying CI, DMP, and SYM, and they are Hypergeometric SLE. For the critical Ising model on the square lattice, the pair of interfaces does converge to Hypergeometric SLE(3). In this talk, we will explain two different proofs for the convergence. Furthermore, we will discuss results about global and local multiple SLEs, which correspond to the scaling limit of the collection of interfaces with alternating b.c. in more general setting.

Asymptotic for critical value of the large-dimensional SIR epidemic on clusters

Xiaofeng Xue

Beijing Jiaotong University

In this talk we are concerned with the SIR (Susceptible-Infective-Removed) epidemic on open clusters of bond percolation on the square lattice. For the SIR model, a susceptible vertex is infected at rate proportional to the number of infective neighbors while an infective vertex becomes removed at a constant rate. A removed vertex will never be infected again. We assume that at $t = 0$ the only infective vertex is the origin and define the critical value of the model as the supremum of the infection rates with which infective vertices die out with probability one, then we show that the critical value under the annealed measure is $\big(1 + o(1)\big)/(2dp)$ as the dimension d of the lattice grows to infinity, where p is the probability that a given edge is open. Furthermore, we show that the critical value under the quenched measure equals the annealed one when the origin belongs to an infinite open cluster of the percolation.

Applications of Arak's inequalities to the Littlewood–Offord problem

A.Yu. Zaitsev

St.Petersburg Department of Steklov Mathematical Institute

Let $X_1, \dots, X_n$ be independent identically distributed random variables. In this talk, we discuss the behavior of concentration functions of weighted sums $\sum_{k=1}^n X_k a_k$ with respect to the arithmetic structure of coefficients a_k in the context of the Littlewood–Offord problem. In the recent paper [1], we studied the relations between the inverse principles stated by Nguyen, Tao and Vu (see [3] and [4]) and similar principles formulated by Arak in his papers from the 1980's. In this talk, we provide some more general and more precise consequences of Arak's inequalities providing new results in the Littlewood–Offord problem, see [2].

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[1] F. Götze, Yu. S. Eliseeva, A. Yu. Zaitsev, Arak inequalities for concentration functions and the Littlewood–Offord problem. Theory Probab. Appl., 62, no. 2 (2017), 241–266 (in Russian). English version: arXiv:1506.09034.

[2] F. Götze, A. Yu. Zaitsev, New applications of Arak's inequalities to the Littlewood–Offord problem. arxiv:1611.00831.

[3] Hoi Nguyen, Van Vu, Optimal inverse Littlewood–Offord theorems. Adv. Math., 226 (2011), 5298–5319.

[4] T. Tao, Van Vu, Inverse Littlewood–Offord theorems and the condition number of random discrete matrices. Ann. of Math. (2), 169 (2009), 595–632.

Correlations between conjugate algebraic numbers: a random polynomial approach

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We describe the asymptotic distribution of all algebraic numbers of fixed degree and large height in terms of correlation functions of a point process of zeros of some random polynomial.

Based on a joint work with Friedrich Goetze and Denis Koleda:

https://arxiv.org/abs/1703.02289

Liouville FPP: geodesic dimension is strictly larger than 1 at high temperatures

Fuxi Zhang

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Let $\{h_v\}$ be a discrete Gaussian free field in a large two-dimensional box with Dirichlet boundary conditions. We investigate the Liouville first passage percolate, i.e. the shortest path metric where each vertex v is given a weight of $e^{\gamma h_v}$ for some $\gamma > 0$. We show that for sufficiently small but fixed $\gamma > 0$, with high probability the dimensions of all geodesics are simultaneously strictly larger than 1. (Joint work with Jian Ding).

Heat kernel estimates and Riesz's transform of non-symmetric non-local operators

Xicheng Zhang

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In this talk I will introduce some new results about the two-sides estimate and Riesz's transform of non-symmetric nonlocal operators. This is a joint work with Zhen-Qing Chen.