Research works by Dr. Dayue Chen

The main topics of Dayue Chen's research works are the follows

Random walk in random environment [6][13][14][15][16][18][19];

The metastability of Markov processes [4][5][7][8][9][11];

Interacting particle systems on trees and graphs [1][2][3][10][12][17].

1. **Random walk in random environment**. By random environment we mean the open cluster of percolation or a Glaton-Watson tree of a branching process. Random walk is the simplest model of process, and serves as a stepping stone for investigating other probability models.

In article [13] we study the percolation model of the Scherk's graph, and prove that the simple random walk is transient if p > 1/2, and recurrent if p < 1/2. This exhibits a phase transition above the critical point which is less than 1/2. The first result in this direction was obtained by Grimmett, Kesten and Zhang in 1993. They showed that the simple random walk on the infinite open cluster of 3-dimensional lattice is transient. This result induces interests in finding graphs which are smaller than 3-dimensional lattice. For example, Angel, Benjamini, Berger and Peres (2004) consider a wedge in the 3-dimensional lattice and obtain the same results. Namely, the simple random walks on the infinite open cluster and the original graph are both recurrent or both transient. My result indicates that the Scherk's graph is so far the only example which possesses another transition above the critical point. The Scherk's graph is a subgraph of the 3-dimensional lattice. In early 90s it was discovered that the simple random walk on the Scherk's graph is transient. This fact is used to prove that the Scherk's surface, as a minimal surface, possesses a non-trivial harmonic function.

Article [16], a joint work with Yuval Peres of Univ. of California, studies the speed of the simple random walk on the open cluster of the Lamplighter groups. For transient random walk, it is natural to study the speed of the simple random walk. Thinking percolation models as a random perturbation of a graph, people wish to know whether positivity of the speed of random walk is unchanged under random perturbations. Benjamini, Lyons and Schramm (1999) verified this statemenrt for graphs with positive Cheeger constant. A more delicate case is that a graph has zero Cheeger constant but grows exponentially. Lamplligher Group G_k is exactly a graph of this type, where k is a positive integer. The speed of the simple random walk on G_k is zero if k = 1 or 2, and is positive if $k \geq 3$. Peres and I studied the simple random walk on the infinite cluster of G_k and showed that positivity of the speed of the simple random walk is unchanged under percolation in this case. Article [16] also discusses the invariance of a geometric quantity, similar to the Cheeger constant, under two random perturbations.

Article [6] proves that the speed of λ-biased random walk on Galton-Watson tree satisfies the following inequality.

$$\frac{m-\lambda}{m+\lambda} \geq \lim_{n\to\infty} \frac{|X_n|}{n} \geq \frac{m'-\lambda}{m'+\lambda} \quad ,$$

where m is the mathematical expectation (arithmetic mean) of offspring distribution of the Galton-Watson process, where m' is the harmonic mean of offspring distribution. The upper bound was conjectured by Lyons, Pemantle and Peres, and was later generalized to any trees by Virag in 2000. In general, lower bounds are more difficult. The above lower estimate is the best up to date.

2. **The metastability of Markov processes.** By metastable behavior we mean some peculiar phenomena observed in a physical system before reaching a stable state. For example, crystal growth in a carefully chosen environment. Research on metastability by mathematical physicists and probabilists has lasted for more than 10 years, and is listed as a direction for future study in the 2003 report by Eurandom.

In articles [8] and [9], a joint work with M.P. Qian and J.F. Feng, we apply the theory of large deviation by Freidlin and Wentzell to prove the metastability of stochastic Ising model on a three dimensional lattice. R. Schonmann studied the 2-dimensional counterpart and formulated some quantitative descriptions into rigorous mathematical theorems. In the 1998 International Congress of Mathematicians he gave a talk entitled “Metastability and the Ising Model”. But his approach could not be used to deal with the 3-dimensional case, which is more interesting in statistical physics. He listed the 3-dimensional Ising model as an open problem. We are able to identify attractors and attractive basins of different levels according to the Hamiltonian of the stochastic Ising model, to find a path with the minimum energy barrier from an attractor of the highest level to an attractor of the second highest level, and to characterize the critical droplet. This problem has been also investigated by scholars from France, Italy, Brazil etc, e.g., Ben Arous of Switzerland. We are certainly independent and among the first in solving the problem, but our results were less propagated.

Articles [7] and [11] address the metastability of the majority vote process. This model is non-reversible, there is no potential function like the Hamiltonian of the Ising model. We introduce the pseudo-potential function. The path with the minimum pseudo-potential barrier from the attractor of the highest level to an attractor of the second highest level is completely different from that of the Ising model. The biased case is more complicated, the path with the minimum barrier delicately depends on the biased parameter. Our findings enrich our understanding of the metastability of Markov processes. Article [11] is co-authored with O. Catoni of France.

Article [5] is devoted to establishing the large deviation theory for metastability of a family of exponentially perturbed Markov processes, and is the corner stone of articles [7],[8],[9] and [11]. The hierarchic structure of attractors of Markov processes, proposed in [5], is an effective way to describe a complex structure, and is similar to the multi-scale method in image processing and scientific computing, and can be applied to a large class of Markov processes. For example, it is used to estimate the convergence rate of simulated annealing. Article [5] is most cited among the work of Dayue Chen.

For this part of research work, Dayue Chen and Minping Qian received an Award (Second Class) for Progress in Science & Technology from the Municipal Government of Beijing.

3. **Interacting particle systems on trees and graphs** were first investigated by the end of 1980s and richer phenomena were reported, for example, the *survival* of the contact process should be classified into two categories: *weak survival* and *strong survival*. This finding stimulated investigation into the relation between the limit behaviors of interacting particle systems and the geometric properties of the underlying graph, and pushed forward the frontier of probability research.

Article [3] defined a nearest particle system on a regular tree, and identified the critical point by the reversibility. The same idea was also used in the Ph.D. dissertation of A. Puha, *A reversible interacting particle system on the homogeneous tree*, directed by T.M. Liggett.

Article [10] discussed the contact process on Galton-Watson trees. We proved that the two critical values are constants, independent of underlying random trees. Under some restriction, two constants are different, namely, there are weak survival and strong survival for the contact process on Galton-Watson trees. This problem was also independently addressed by R. Pemantle and R. Stacy.

My interests in this direction are also reflected in the dissertations/theses supervised by me. For example, Shumei Jia, my first Ph. D. student, submitted in 2002 a dissertation entitled *the contact process on transitive graphs*. Dr. Jia has published two papers on this subject. Other theses for Master of Science include:

The limit behavior of the contact process on Galton-Watson tree by Lijiao Zhao (1998)
The limit behavior of the biased contact process on a regular tree by Shumei Jia (1999)
A study of a mixed model on $T_d \times Z$ by Nan Chen (2001)
The reversible nearest particle system on an interval by Juxin Liu (2003)
A study on L2 convergence of stochastic Ising model by Lie Wang (2003)
The contact process on planar graphs by Shimo Wang (2004)
The contact process and branching random walk on stretched trees by He Li (2005)

Four of them continue their studies leading to Ph.D. in China, Canada and the U.S..

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